Matrices

MATRIX: A MATRIX IS AN ORDERED RECTANGULAR ARRAY OF NUMBERS OR FUNCTIONS. THE NUMBERS OR FUNCTIONS ARE CALLED THE ELEMENTS OR THE ENTRIES OF THE MATRIX.

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School

Divya
N – 10
P – 5
C – 6

Daksh
N – 8
P – 1
C – 3

OM
N – 4
P – 9
C – 8

$$A = \begin{array}{c|c} & \begin{array}{ccc} N & P & C \end{array} \\ \begin{array}{c} D \\ D \\ O \end{array} & \begin{bmatrix} 10 & 5 & 6 \\ 8 & 1 & 3 \\ 4 & 9 & 8 \end{bmatrix} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \\ & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array}$$

Matrix

Rectangular array

Rows

Columns

elements / entries of Matrix A

Total No. of Rows = 3

" " " Colums = 3

R_3

$B = [\quad]$

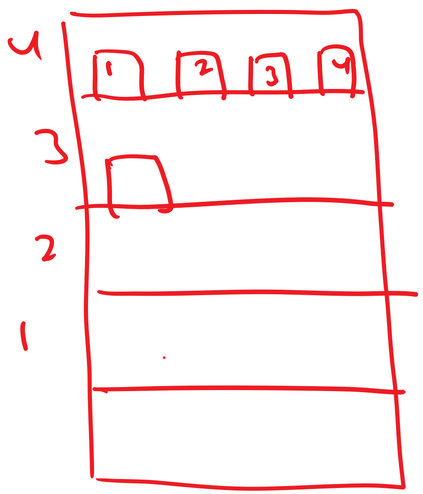
Order of a Matrix

Ex :

$$A = \begin{bmatrix} 1 & -3 & 4 & 2 \\ 9 & 1 & 3 & 0 \\ 2 & 5 & 6 & 4 \end{bmatrix}$$

elements

Order = ?



$m \times n$

T.N.O Rs ← $\boxed{3 \times 4}$ → TNO Cs

3 cross 4

3 by 4

12 → T.N.O. elements

$$\boxed{\text{TNO Es} = mn}$$

Equality of Matrices

Condition: → order Same.

→ Corresponding element

$$\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$$

Ex: If $\begin{bmatrix} x & 3 \\ 2 & y \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}_{2\times 2}$

$x = ?$

$y = ?$

$\boxed{x = 1}$

$\boxed{y = 4}$

Ex If $\begin{bmatrix} x+y & 4 \\ 3 & x-y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$

$x = ?$

$y = ?$

$x = 3$

$y = -2$

Elements Notation

Ex:

$$B = \begin{bmatrix} 3 & 4 & -2 \\ 1 & 9 & 8 \end{bmatrix}_{2\times 3}$$

a_{11} a_{13}

place?

a_{22} a_{23}

a_{ij} → Column place

element

Row place

Q. find

$$\frac{a_{21} + (a_{13})^2}{2}$$

$$\frac{1 + (-2)^2}{2}$$

$$\frac{5}{2}$$

Is $m = i$?

Is $n = j$?

Is $m \neq i$? T

Is $n \neq j$? T

$m = i$

m → TNORs

i → Row place

$\Rightarrow \quad A = [a_{ij}]_{m \times n}$

A → Matrix

a → element

i → Row place

j → Column place.

m → T.NORs

n → TNOCs

$$A = [a_{ij}]_{m \times n}$$

$\downarrow$ define

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2n} \\ a_{31} & \vdots & \vdots & \vdots & & \vdots \\ a_{41} & \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & \cdots & \cdots & \boxed{a_{mn}} \end{bmatrix}_{m \times n}$$

Ex : $A = [a_{ij}]_{3\times3}$

define

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

Q: Matrix — A

Total elements = 6

→ possible No. of orders = ?

How Many? and list them.

A = [6 element]

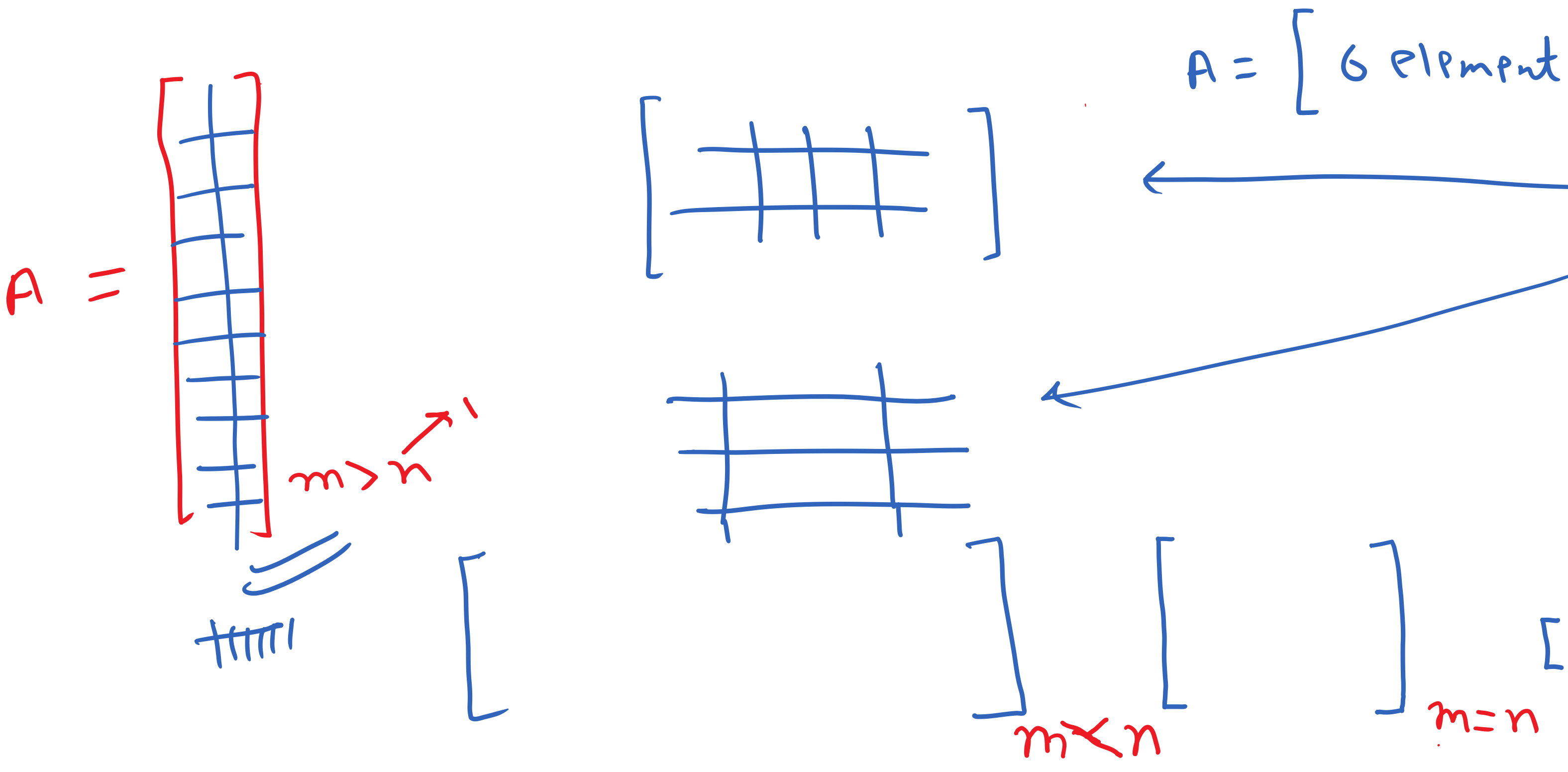
1×6

2×3

3×2

6×1

4 ways



$m < n$

Q: Construct Matrix $A = [a_{ij}]_{2\times 2}$

$a_{ij} = \frac{i+j}{2}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

$a_{11} = \frac{1+1}{2} = \frac{2}{2} = 1$

$i \quad j$

$a_{12} = \frac{1+2}{2} = \frac{3}{2}$

$a_{21} = \frac{3}{2}$

$a_{22} = \frac{4}{2} = 2$

$$A = \begin{bmatrix} 1 & 3/2 \\ 3/2 & 2 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

$$x+y+z=9 \qquad x+z=5 \qquad y+z=7$$

$$5+y=9$$

$$\boxed{y=4}$$

$$x+7=9$$

$$\boxed{x=2}$$

$$2+4+z=9$$

$$z=9-6$$

$$\boxed{z=3}$$

Basic Concept

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & 9 \\ 8 & 3 & 4 \end{bmatrix}$$

diagonal 1, 2, 4 → diagonal elements

$i = j$

Non - diagonal element $i \neq j$

$i < j$
$1 < 2$

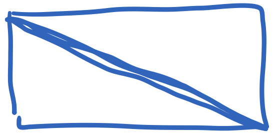
$i < j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$i > j$

$3 > 2$
$i > j$

Padhui

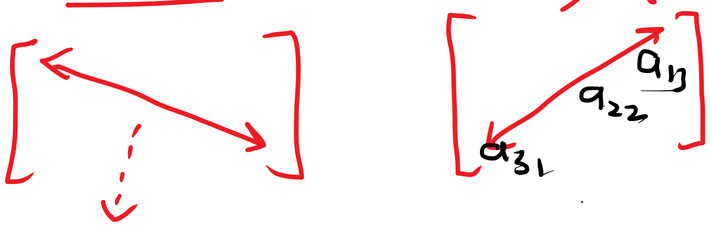


* diagonal elements $i = j$
* Non - diagonal elements $i \neq j$

above the diagonal elem $\boxed{i < j}$

Below the diagonal ele $i > j$

Matrix



Types of Matrix

Row Matrix

$m = 1$

$A = [a_{ij}]_{1 \times n}$

A matrix of only one Row, no limit of Column.

Ex: $A = \begin{bmatrix} 1 & 2 & 4 \end{bmatrix}_{1 \times 3}$

only one Row

Column Matrix

$n = 1$

$B = [b_{ij}]_{m \times 1}$

A matrix of only one column, no limit of Row

Ex: $B = \begin{bmatrix} 3 \\ 9 \\ 6 \end{bmatrix}_{3 \times 1}$

only one Column

Zero Matrix

Null Matrix | Empty Matrix | Void Matrix

$O = [c_{ij}]_{m \times n}$

$\boxed{c_{ij} = 0}$

Ex $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2 \times 3}$

all elements of a Matrix are zero are called zero Matrix

✱✱ Square Matrix

$D = [d_{ij}]_{m \times n}$

If $\boxed{m = n}$ Square Matrix

Ex $D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$

$E = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 2 & 1 & 5 \end{bmatrix}_{3 \times 3}$

Square Matrix $\boxed{m=n}$

Diagonal Matrix | Scalar Matrix | I_n = Identity Matrix / Unitary Matrix | Triangular Matrix

$n \rightarrow$ order

Triangular Matrix: UTM, LTM

Diagonal Matrix

A square Matrix A

$\rightarrow A = [a_{ij}]_{m \times n}$ $\boxed{m=n}$

$\boxed{a_{ij} = 0}$ If $(i \neq j) \rightarrow$ N.D

$a_{ij} = K$ If $\boxed{i = j} \rightarrow$ Diagonal ele.

Ex

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 8 \end{bmatrix} \rightarrow \text{DM}$$

$$\cancel{C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}$$

Scalar Matrix

A Square Matrix

$B = [b_{ij}]_{m \times n}$

$m = n$

&

Ex: $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Same but not 1

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Identity Matrix

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

$\downarrow$

I_3

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q Types of Matrix

(i) $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}_{2\times 2}$ — SM, DM

(ii) $B = \begin{bmatrix} 9 & 0 \\ 4 & 3 \end{bmatrix}_{2\times 2} \rightarrow$ SM

(iii) $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ ZM

(iv) $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ → DM, → SM

(v) $E = \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 9 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$ SM

(vi) $F = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 5 \end{bmatrix} \rightarrow$ DM

(vii) $G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

#

$A = [a_{ij}]_{2\times 2} \rightarrow 0 \,\&\, 1$

Possible No. of Matrix = $(\text{No. of entries})^{mn}$

$= (2)^{2\times 2}$

$= 2^4$

$= \underline{\underline{16}}$

Ex $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

⋮

16?

$\begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 7 \\ 4 & 3 & 5 \end{bmatrix}$

Operations on Matrices

Addition of Matrices

$A = [a_{ij}]_{m \times n} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = [b_{ij}]_{m \times n} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$A + B \rightarrow$ order must be Same

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$\begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}$ Ans

$\Rightarrow \boxed{A + B = B + A}$

Sub. of Matrices

$A = [a_{ij}]_{m \times n} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = [b_{ij}]_{m \times n} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$A - B \rightarrow$ order must be Same

$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix}$

✩✩ Multiplication

Multiplication

Scalar Multiplication

Ex

$$A = \begin{bmatrix} 2 & 3 \\ 4 & -8 \end{bmatrix}$$

$$4A = ?$$

Scalar

$$4A = 4\begin{bmatrix} 2 & 3 \\ 4 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 12 \\ 16 & -32 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$AB = ?$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 4 + 2\times 2 & 1\times 1 + 2\times 3 \\ 3\times 4 + 4\times 2 & 3\times 1 + 4\times 3 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 7 \\ 20 & 15 \end{bmatrix}$$

1 $\boxed{2}$ → 1st Matrix (A)

4 $\boxed{2}$ → 2nd Matrix (B)

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \end{bmatrix}_{2\times 3} \quad B = \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}_{3\times 2}$

If → NOC of 1st = NOR of 2nd then Mutiply poss.

$AB = ?$

Step-1

$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$

Step-2

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1\times 3+2\times 2+3\times 1 & 1\times 0+2\times 2+3\times 4 \\ 4\times 3+5\times 2\times 2\times 1 & 4\times 0+5\times 2+2\times 4 \end{bmatrix}$

$= \begin{bmatrix} 10 & 14 \\ 24 & 18 \end{bmatrix}_{2\times 2}$

$BA = ?$

$BA = \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 1 & 4 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \end{bmatrix}_{2\times 3}$

$= \begin{bmatrix} 3 & 6 & 9 \\ 10 & 14 & 10 \\ 17 & 22 & 11 \end{bmatrix}$ Ans

3×3

Q

$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}_{2\times 2}$ $\quad B = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

$AB = ?$

☆ $BA = ?$ $\quad \begin{bmatrix} 3 & 1 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ \boxed{} & \end{bmatrix}_{2\times 2}$

$AB = \begin{bmatrix} 14 & 18 & 34 \\ 9 & 13 & 22 \end{bmatrix}_{2\times 3}$

Q There are four matrices A, B, C & D, orders are $m \times n$, $2 \times p$, $3 \times q$, $x \times 4$

(i) If order of AB is 3×2 find $m = ?$, $n = ?$, $p = ?$

$$AB = \begin{bmatrix} \; \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} \; \end{bmatrix}_{m \times n} \cdot \begin{bmatrix} \; \end{bmatrix}_{2 \times p} = \begin{bmatrix} \; \end{bmatrix}_{3 \times 2}$$

$n = 2$

3×2

$m \times p$

$m = 3$

$p = 2$

$n = 2$

(ii) If $C + D$ is possible then find $x = ?$, $q = ?$

$$\begin{bmatrix} \; \end{bmatrix}_{3 \times q} + \begin{bmatrix} \; \end{bmatrix}_{x \times 4}$$

$x = 3$

$q = 4$

Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. Choose the correct answer in Exercises 21 and 22.

21. The restriction on n, k and p so that PY + WY will be defined are:

$$[\ ]_{p \times k} [\ ]_{3 \times k} + [\ ]_{n \times 3} [\ ]_{3 \times k}$$

$\boxed{k = 3}$

$$[\ ]_{p \times k} + [\ ]_{\cancel{n} \times k}$$

for addition order must be same $\boxed{p = n}$

22. If n = p, then the order of the matrix 7X – 5Z is:

7 (ii) $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \Big\} \times 3$ — 1

$3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \Big\} \times 2$

$$\cancel{6x} + 9y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$$

$$\cancel{6x} + 4y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$5y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$y = \frac{1}{5} \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix} \qquad y = \begin{bmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{bmatrix}$$

$$2x + 3 \begin{bmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$2x + \begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -6 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$2x = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -6 \end{bmatrix}$$

$$2x = \Big[\; 4/5$$